Problem 1.13

The charge entering the positive terminal of an element is

$$q = 5 \sin 4\pi t \text{ mC}$$

while the voltage across the element (plus to minus) is

$$v = 3 \cos 4\pi t \text{ V}$$

(a) Find the power delivered to the element at $t = 0.3$ s.

(b) Calculate the energy delivered to the element between 0 and 0.6 s.

Solution

Part (a)

The power is the product of voltage and current.

$$\begin{aligned} p(t) = v(t)i(t) = v(t)\frac{dq}{dt} &= (3 \cos 4\pi t \text{ V})\frac{d}{dt}(5 \sin 4\pi t \text{ mC}) \\ &= (3 \cos 4\pi t \text{ V})(20\pi \cos 4\pi t \text{ mA}) \\ &= 60\pi \cos^2 4\pi t \text{ mW} \end{aligned}$$

Therefore, the power delivered to the element at $t = 0.3$ s is

$$p(0.3) = 60\pi \cos^2 \frac{6\pi}{5} \text{ mJ} \approx 123 \text{ mJ}.$$

Part (b)

Integrate the power from $t = 0$ to $t = 0.6$ s to find the energy delivered to the element in this interval.

$$\begin{aligned} W = \int_0^{0.6} p(t)\,dt = \int_0^{0.6} (60\pi \cos^2 4\pi t)\,dt \text{ mW} &= 60\pi \int_0^{0.6} \cos^2 4\pi t\,dt \text{ mW} \\ &= 60\pi \int_0^{0.6} \frac{1}{2}(1 + \cos 8\pi t)\,dt \text{ mW} \\ &= 30\pi \left(t + \frac{1}{8\pi} \sin 8\pi t \right)\Big|_0^{0.6} \text{ mJ} \\ &= 30\pi \left(\frac{3}{5} + \frac{1}{8\pi} \sin \frac{24\pi}{5} \right) \text{ mJ} \\ &\approx 58.8 \text{ mJ} \end{aligned}$$